An arithmetic theory of local constants

B. Mazur's lecture for LP

May 21, 2006

Abstract

There is a well-working, and classical, theory of ϵ-factors attached to local Galois representations (due to Langlands, Deligne, and others) these being the quantities that are conjectured to enter into the functional equations that are expected to be satisfied by L-functions of motives.

In an appropriate context, when the Galois representation in question is equal to its contragedient, and the ϵ-factors are ± 1, the "sign" of the functional equation (and hence the parity of the order of vanishing of the L-function at the central point) is dictated by the values of these knowledge of these ϵ-factors. All of this, however, depends on a cascade of conjectures.

Karl Rubin and I have begun to develop a corresponding arithmetic theory of ϵ-factors—these being easily computed in terms of Galois cohomology, but follow (as far as we have developed the theory) the exact pattern of the analytic theory of ϵ factors, and give information about changes of parity of the Selmer group of an elliptic curve over a number field when twisted by different finite characters. As a corollary we obtain a self-contained proof, by a new method, of a significant generalization of previous results that guarantee large Selmer rank.