Character sheaves and the orbit method

Abstract

The talk will be based on our recent joint work with Vladimir Drinfeld on certain geometric aspects of character theory for unipotent groups over finite fields.

Let F_q be a finite field with q elements, where q is a power of a prime number p, let F be a fixed algebraic closure of F_q, and let G be a connected unipotent group over F whose nilpotence class is less than p. We define a certain collection of irreducible perverse sheaves on G, which we call the character sheaves, that enjoy the following property.

Suppose G comes from a unipotent group G_0 over F_q, and let $\mathrm{Frob} : G \to G$ denote the corresponding Frobenius endomorphism, obtained from the absolute Frobenius of G_0 by extension of scalars. For each character sheaf M on G such that $\mathrm{Frob}^*(M)$ is isomorphic to M, let us fix one such isomorphism, and let f_M denote the function on $G_0(F_q)$ obtained from this isomorphism via the functions-sheaves dictionary. Then the functions f_M form a basis for the space of class functions on the finite group $G_0(F_q)$. In addition, there exist explicit formulas expressing irreducible characters of $G_0(F_q)$ as linear combinations of the functions f_M.

Thus our theory is similar in spirit to Lusztig's theory of character sheaves for reductive groups over finite fields.

The proof of the main result stated above crucially depends on Kirillov's orbit method. (However, the definition of character sheaves does not depend on it.)

In my talk I will:

(i) recall the main points of the classical orbit method;

(ii) give the definition of character sheaves for unipotent groups and explain their relation to irreducible characters (when the nilpotence class is less than the characteristic of the ground field);

(iii) explain the geometric phenomena arising in the characteristic p theory that make it different from and more interesting than the classical orbit method.