Yangians and Mickelsson algebras

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Let $\mathfrak{H}_N$ be the degenerate affine Hecke algebra corresponding to the group GL_N over a p-adic field. There are two well known functors in the representation theory of $\mathfrak{H}_N$. One of them, first introduced by Cherednik and then studied by Arakawa, Suzuki and Tsuchiya, is a functor from the category of modules over the Lie algebra $\mathfrak{gl}_m$ to the category of $\mathfrak{H}_N$-modules. The other, introduced by Drinfeld, is a functor from the latter category to the category of modules over the Yangian $Y(\mathfrak{gl}_n)$ of the Lie algebra $\mathfrak{gl}_n$. This Yangian is a deformation of the universal enveloping algebra of the polynomial current Lie algebra $\mathfrak{gl}_n[t]$ in the class of Hopf algebras.

The composition of these two functors is particularly important. It provides a representation theoretic explanation of the correspondence between the "extremal cocycle" on the Weyl group of $\mathfrak{gl}_m$ defined by Zhelobenko, and the intertwining operators between tensor products of $Y(\mathfrak{gl}_n)$-modules. This correspondence, first established by Tarasov and Varchenko, involves the classical dual pair of Lie groups (GL_m, GL_n) and is reviewed in this talk. The main aim of the talk is to introduce an analogue of the composition of the two functors, when the pair (GL_m, GL_n) is replaced by any of the dual pairs (Sp_{2m}, O_n) or (SO_{2m}^*, Sp_n) introduced by Howe. The role of the Yangian $Y(\mathfrak{gl}_n)$ is then played respectively by the twisted Yangians $Y(\mathfrak{so}_n)$ or $Y(\mathfrak{sp}_n)$. These twisted Yangians are coideal subalgebras of $Y(\mathfrak{gl}_n)$, and deformations of the universal enveloping algebras of the twisted polynomial current Lie algebras

$$\{\, X(t) \in \mathfrak{gl}_n[t] \mid \sigma(X(t)) = X(-t) \,\}$$

where σ is the involutive automorphism of $\mathfrak{gl}_n$ with the fixed point subalgebra $\mathfrak{so}_n$ or $\mathfrak{sp}_n$ respectively. This talk is based on my joint works with Sergey Khoroshkin.